EC 709: Monotonicity and 2SLS

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Overview

1. A test for IV validity
2. A Classic Example
3. General 2SLS and LATE

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Review of Assumptions

A1. Instrument Relevance: $Pr(D=1|Z=1) \neq Pr(D=1|Z=0)$

- Can be assessed by inferring the coefficient in the first-stage regression: Convention view, $F \geq 10$
- New researches show this is not enough (See Lee et al (2022) and Keane and Neal (2023))

A2. Exclusion restriction contains two parts:

1. No direct effect on potential outcome: With probability 1, $Y_{d1} = Y_d|Z=1 = Y_d|Z=0 = Y_{d0}$ for $d = 1, 0$
2. Random Assignment: The variable Z is jointly independent of $(Y_{11}, Y_{10}, Y_{01}, Y_{00}, D_1, D_0)$

$\Rightarrow$ Z affect D only by affecting whether the treatment is more likely to be D_1 or D_0

A3. Monotonicity: With probability 1, the potential treatment response indicators satisfy $D_{1i} \geq D_{0i} \forall i$ or $D_{0i} \geq D_{1i} \forall i$

? How do we test A2 and A3 in practice?

A Necessary Condition on IV validity

- Start with something observable:

$$
\begin{aligned}
&Pr(Y = y, D = 1|Z = 1) - Pr(Y = y, D = 1|Z = 0) \\
&= (Pr(Y_1 = y, D_1 = 1, D_0 = 0|Z = 1) + Pr(Y_1 = y, D_1 = 1, D_0 = 1|Z = 1)) \\
&- (Pr(Y_1 = y, D_1 = 1, D_0 = 1|Z = 0) + Pr(Y_1 = y, D_1 = 0, D_0 = 1|Z = 0)) \\
&\text{(By Random assignment of Z to potential outcomes:)} \\
&= Pr(Y_1 = y, D_1 = 1, D_0 = 0|Z = 1) - Pr(Y_1 = y, D_1 = 0, D_0 = 1|Z = 0) \\
&= Pr(Y_1 = y, D_1 = 1, D_0 = 0) - Pr(Y_1 = y, D_1 = 0, D_0 = 1)
\end{aligned}
$$

- Further assuming $D_{1i} \geq D_{0i} \forall i$ and by the similar procedure:

$$
\begin{aligned}
Pr(Y = y, D = 1|Z = 1) - Pr(Y = y, D = 1|Z = 0) = Pr(Y_1 = y, D_1 > D_0) \\
Pr(Y = y, D = 0|Z = 0) - Pr(Y = y, D = 0|Z = 1) = Pr(Y_0 = y, D_1 > D_0)
\end{aligned}
$$

$\Rightarrow$ Testable implication: nonnegative difference in densities

A Test For Instrument Validity (Kitagawa, 2015)

$p(y, d) = Pr(Y = y, D = d|Z = 1)$, $q(y, d) = Pr(Y = y, D = d|Z = 0)$

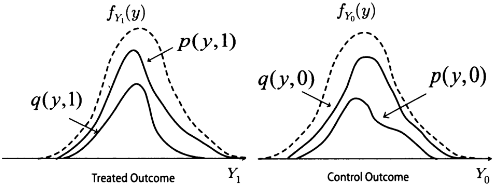

Figure 1: IV validity cannot be refuted

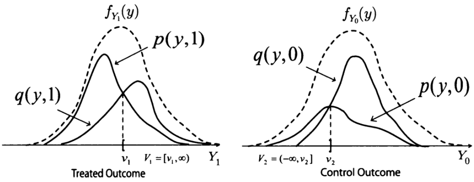

Figure 2: Can refute at least one of the IV validity assumptions

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Card (1995)

- Data: 24-year-old men from the 1976 interview of the NLSYM, $N = 3,010$
  - National Longitudinal Survey of Young Men (NLSYM): sampled men aged 14–24 in 1966 and continued with follow-up surveys through 1981
- Question: estimate the returns to education
  - The outcome Y: log hourly wage; The treatment D: indicates whether one graduated from a four-year college
  - Endogenity: Omitted ability measure
- The instrument Z: a binary indicator for the presence of an accredited four-year college in the local labor market when the respondent was 14 years old
  - "the distance to the nearest college" as an instrument for educational attainment
  - First Stage: Live in an area close to college makes students more likely to attend college

  $\Leftarrow$ the presence of a nearby college reduces the cost of college education by allowing students to live at home

Testing For Card (1995)

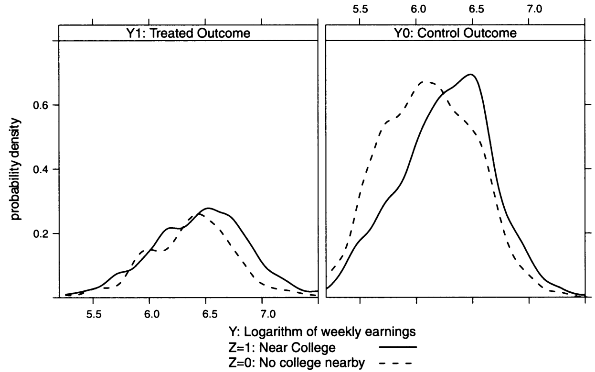

FIGURE 4.—Kernel density estimates: proximity to college data. The Gaussian kernel with bandwidth 0.08 is used.

- Either Exclusion restriction or Monotonicity failed

$\Rightarrow$ Need to diagnose by economics intuition and incorporate more general methods

Problem of the Exclusion restriction

- Implication of A2: Student's unobservable ability is presumably independent of students' residence during their teenage years
  - Do not control for any demographic covariates

  $\Rightarrow$ Raises a concern regarding the violation of the random assignment assumption

  E.g., Urban areas are more likely to have colleges and higher wage levels compared to the Rural areas
- After adding some additional covariates, Kitagawa (2015) failed to reject the validity of Card (1995)'s instrument:
  - Five binary variables: whether Black, whether lived in a metropolitan area (SMSA) in 1966 and 1976, and whether lived in the South in 1966 and 1976
- In Card's main results (1995, Table 3A, column (5)), he indeed emphasized the importance of adding additional controls
- Based on a survey by Blandhol et al (2022) , 81% of papers using IV included at least one covariate X

Problem of the Monotonicity

- Compilers: Students grew up in relatively low-income families and who were not able to go to college without living with their parents
- Frolich and Sperlich (2019): Might not be the only direction
  - Some students may be encouraged to attend college if the nearest college is far away, as this gives them an excuse to move out of the parental home

  $\Leftarrow$ Defier: "defy" their instrument assignment for any reason
- Monotonicity assumption is likely to fail

Solution: Weaker the Monotonicity

- Weak Monotonicity (WM): There exists a partition of the covariate space such that $P[D_1 \geq D_0|X] = 1$ a.s. on one subset and $P[D_1 \leq D_0|X] = 1$ a.s. on its complement
  - Defiers we mentioned above are unlikely to affect students with binding financial constraints
  - It is conceivable that college proximity never discourages poor students from attending college and never encourages rich students to do so

  $\Rightarrow$ Consistent with the Weak Monotonicity with the partition on the income level of the households

  $\Leftarrow$ Direction itself is allowed to be different for the two groups
- Obviously true that Assumption WM is weaker than Monotonicity
- Still restrictive to assume that all rich students and all poor students are affected by college proximity in the same direction (if at all)

Practical Issues in the General Cases

- In practice, we often need to **add some covariates** and **assume weak monotonicity** to achieve a valid IV
- Are we still estimating LATE in our 2SLS with these further generalizations?

$\Rightarrow$ I will offer a quick review of what is trending in the IV literature

! Make sure the covariates you add are exogenous!

$\Leftarrow$ Glynn and Rueda (2017) called that post-instrument bias if covariates are itself endogenous

$\Rightarrow$ OLS with an omitted variable will often have less bias than IV with the post-instrument covariate

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Covariates + Monotonicity

- X: a vector of control variables including a constant; $P[D_1 \geq D_0] = 1$
- $\beta_{iv} = \frac{E[Y\tilde{Z}]}{E[T\tilde{Z}]}$, where $\tilde{Z} = Z - L[Z|X]$
    - $\tilde{Z}$: residuals from a regression of Z on X
    - $L[Z|X] = X^{\intercal}E[XX^{\intercal}]^{-1}E[XZ]$: instrument propensity score when Z is binary
- Blandhol et al (2022) shows:

$$\beta_{iv} = E[\omega(cp, X)\tau(cp, X)] + E[\omega(at, X)\tau(at, X)] + E[\omega(nt, X)\tau(nt, X)]$$

- $\tau(T, X)$: conditional average treatment effects for group T; $\omega(T, X)$: weights on group T
- Whenever $L[Z|X] \neq E[Z|X]$, the IV estimand incorporates negatively weighted treatment effects for some groups

Covariates + Weak Monotonicity

- Now given that $L[Z|X] = E[Z|X]$, when we only assumed the Weak Monotonicity :
- Słoczynski (2022) shows there would also be negative weights

$\Leftarrow$ Reduced-form and first-stage regressions implicitly restrict the effects of the instrument to be homogeneous and are thus possibly misspecified

- Even if all weights are positive, the IV estimand in the just identified specification is not interpretable as the unconditional LATE
  - $\Leftarrow$ when almost no individuals are encouraged to get treated, the IV estimand is similar to the local average treatment effect on the treated
  - $\Leftarrow$ The opposite of what we want if our goal is to recover the unconditional LATE parameter
  - Słoczynski (2020) finds the similar phenomena for OLS as well

Link back to Card (1995)

- Both papers use the Card(1995) for empirical application
- Use 2SLS, college attendance yields earnings gains of about 60 log points,
  - Outside the range of estimates in the recent literature
  - $\Leftarrow$ Driven by the presence of negative weights
  - Corrected estimates indicate that attending college causes earnings to be roughly 20% higher

Issues of IV with covariates

1. Misspecification of the model for the instrument propensity score could lead to a large bias
2. 2SLS implicitly restricts the effects of the instrument to be homogeneous
3. Not desired weight for unconditional LATE

$\Rightarrow$ Motivate the use of matching techniques

Thank You!